Erratum: Unified solution of the inverse capacity problem [Phys. Rev. E 57, 1302 (1998)]

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Equation (20) should read

$$g(\nu)=\frac{2}{rk\nu}\sum_{m=2}^{\infty}\frac{a_{2m-1}(h\nu/k)^{2m-1}}{(-1)^{m+1}(2\pi)^{2m}B_{2m}}. \tag{20}$$

Equation (21) should read

$$g(\nu)=\frac{2}{rk\nu}\frac{(-1)^{2+1}a_3(h\nu/k)^3}{(2\pi)^4B_4}=\frac{2}{rk\nu}\frac{(-1)^{2+1}a_3(h\nu/k)^3}{(2\pi)^4(-1/30)}=\frac{15a_3h^3}{4\pi^4k^4r}\nu^2. \tag{21}$$

The deduction of (B13) and (B14) should be

$$1=\frac{-\Gamma(z)}{2\pi i}\int_{\infty}^{(0+)}e^{-t}(-t)^{-z}dt=\frac{-\Gamma(m+1)}{2\pi i}\int_{\infty}^{(0+)}e^{-(n+1)x}[-x(n+1)]^{-(m+1)}(n+1)dx$$
$$=\frac{-m!}{2\pi i}\int_{\infty}^{(0+)}e^{-(n+1)x}(n+1)^{-m}(-x)^{-(m+1)}dx. \tag{B13}$$

$$\rightarrow(n+1)^m=\frac{-m!}{2\pi i}\int_{\infty}^{(0+)}e^{-(n+1)x}(-x)^{-(m+1)}dx. \tag{B14}$$

Then,

$$\lim_{0<y\rightarrow 1^{-1}}\sum_{n=0}^{\infty}(n+1)^m y^{n+1}=\frac{-m!}{2\pi i}\int_{\infty}^{(0+)}(-x)^{-(m+1)}\sum_{n=0}^{\infty}e^{-(n+1)x}dx$$
$$=\frac{-m!}{2\pi i}\int_{\infty}^{(0+)}(-x)^{-(m+1)}e^{-x}\sum_{n=0}^{\infty}e^{-nx}dx$$
$$=\frac{-m!}{2\pi i}\int_{\infty}^{(0+)}(-x)^{-(m+1)}\frac{e^{-x}}{1-e^{-x}}dx. \tag{B15}$$

The deduction of (B19) should read as follows.

The integration in right-hand side has a pole of rank $(m+2)$ at $x=0$, thus

$$\frac{1}{2\pi i}\int_{\infty}^{(0+)}(-x)^{-(m+1)}\frac{e^{-x}}{1-e^{-x}}dx=\mathrm{Res}_{x=0}\left[(-x)^{-(m+1)}\frac{e^{-x}}{1-e^{-x}}\right]$$
$$=\mathrm{Res}_{x=0}\left[(-x)^{-(m+1)}\frac{1}{e^x-1}\right]$$
$$=\frac{(-1)^{m+1}}{(m+1)!}\lim_{x\rightarrow 0}\frac{\partial^{(m+1)}}{\partial x^{(m+1)}}\left[\frac{x^{m+2}x^{-(m+1)}}{e^x-1}\right]$$
$$=\frac{(-1)^{m+1}}{(m+1)!}\lim_{x\rightarrow 0}\frac{\partial^{(m+1)}}{\partial x^{(m+1)}}\left[\frac{x}{e^x-1}\right]. \tag{B16}$$

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Based on the generating function of Bernoulli numbers B_n

$$\frac{t}{e^t-1}=\sum_{n=0}^{\infty}\frac{t^n}{n!}B_n, \tag{B17}$$

we have

$$\begin{aligned}\mathrm{B}(16)&=\frac{(-1)^{m+1}}{(m+1)!}\frac{\partial^{(m+1)}}{\partial x^{(m+1)}}\left[\sum_{n=0}^{\infty}\frac{x^n}{n!}B_n\right]\Bigg|_{x=0}\\&=\frac{(-1)^{m+1}}{(m+1)!}\frac{\partial^{(m+1)}}{\partial x^{(m+1)}}\left[\frac{x^{m+1}}{(m+1)!}B_{m+1}\right]=\frac{(-1)^{m+1}}{(m+1)!}\left[\frac{(m+1)!}{(m+1)!}B_{m+1}\right]=(-1)^{m+1}\left[\frac{B_{m+1}}{(m+1)!}\right].\end{aligned}$$

Therefore, it is given that

$$\begin{aligned}\lim_{0<x\to 1^{-1}}\sum_{n=0}^{\infty}(n+1)^m x^{n+1}&=-\frac{m!}{2\pi i}\int_{\infty}^{(0^+)}(-x)^{-(m+1)}\frac{e^{-x}}{1-e^{-x}}\,dx\\&=-m!\left[\frac{1}{2\pi i}\int_{\infty}^{(0^+)}(-x)^{-(m+1)}\frac{e^{-x}}{1-e^{-x}}\,dx\right]=(-m!)\left[(-1)^{m+1}\frac{B_{m+1}}{(m+1)!}\right]=\frac{(-1)^m B_{m+1}}{m+1}.\end{aligned} \tag{B18}$$

Therefore, for any natural number m,

$$\lim_{0<x\to 1^{-}}\sum_{n=0}^{\infty}(n+1)^m x^{n+1}=\lim_{0<x\to 1^{-}}\sum_{n=1}^{\infty}n^m x^n=(-1)^m\frac{B_{m+1}}{m+1}=\begin{cases}0, & m=2k>0,\\ -\dfrac{B_{2k}}{2k}=\zeta(1-2k) & m=2k-1>0.\end{cases}$$

Notice that $B_{2k-1}=0$. Thus the first theorem is proved as

$$\lim_{0<x\to 1^{-}}\sum_{n=1}^{\infty}n^m x^n=\begin{cases}0, & m=2k>0,\\ \zeta(1-2k), & m=2k-1>0.\end{cases} \tag{B19}$$